

MAINTAINING 10 VDC AT 0.3 PPM OR BETTER IN YOUR LABORATORY

Ray Kletke
Fluke Corporation
Everett, Washington

Abstract

There is a need today to maintain dc voltage at an uncertainty of 0.3 ppm or better. We believe that most standards laboratories can do this without using an expensive Josephson Junction Array System. DC Reference Standards commonly in use today are capable of this performance providing that they are properly calibrated and that the effects of temperature, pressure and seasons are taken into account. This paper develops supporting uncertainty equations and applies them to useful scenarios.

Introduction

High end multifunction calibrators and digital voltmeters today require calibration uncertainties that only primary standards laboratories maintained in recent history. Uncertainties of 1.0 to 1.5 ppm are required for 10 VDC at the time of test. These standards, in turn, must be supported by reference standards having a NIST traceable uncertainty of 0.3 to 0.5 ppm if a reasonable Test Uncertainty Ratio (TUR) is to be maintained.

It is possible today to maintain 10 VDC at 0.3 ppm or better in most standards laboratories without a Josephson Junction Array. However, care must be taken to minimize the effects of certain stimuli that usually contributes negligible error and, therefore, is overlooked. This paper examines some of those effects and recommends how they can be controlled so as to achieve 0.3 ppm performance or better.

Classical Approach

The classical approach for estimating the uncertainty of the dc voltage standard is simply to combine its stability, as specified by the manufacturer, with the uncertainty of the calibration as follows:

$$U_{tot} = \sqrt{stab^2 + U_{cal}^2}$$

Modern zener type dc voltage standards typically have a stability of 2 ppm per year. Assuming this value and a calibration uncertainty of 0.1 ppm, the total uncertainty can be calculated as a function of time. Figure 1 gives the total uncertainty for a single cell and triple cell dc voltage standard (DCVS) as a function of its calibration cycle in months. It shows that the 0.3 ppm goal can be approached for the single cell DCVS only if it is calibrated with a Josephson Junction Array System every two months or less. This may

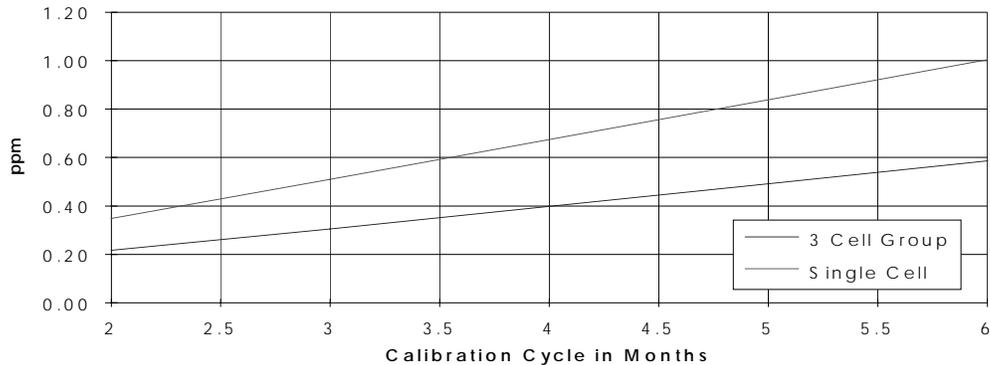


Figure 1. Total Uncertainty for DCVS Array Voltage Standard

or may not be practical depending on the turn-around time of the calibration supplier. If the number of independent cells is increased to three, the required calibration cycle time to maintain 0.3 ppm is increased to 3 months. Although this may be workable, it is inefficient and costly, requiring a total of 12 calibrations per year (4 on each cell).

Characterized Performance

The largest component of uncertainty in the above calculations is the stability of the standard. You might postulate that the typical performance for a standard must be better than its specifications since the manufacturer must achieve a high yield in his manufacturing process. This in fact is true. Therefore, if the standard is characterized using historical calibration data, its actual performance will usually be better than its stability specifications.

A linear regression model is frequently used for this characterization. Historical calibration data is used to calculate the parameters of the regression line and the components of the estimated regression uncertainty. The output voltage of the standard and its uncertainty is given by the following equation:

$$V_{std} = a + bX \pm U \quad (1)$$

where

- a is the regression offset term
- b is the regression slope (drift rate)
- X represents time or date
- U is the total uncertainty of the voltage standard

and
$$U = t(1 - \alpha, v_{eff}) \sqrt{u_{reg}^2 + u_{cal}^2} \quad (2)$$

where t is the t statistic
 $1 - \alpha$ is the confidence factor for t
 v_{eff} is the effective degrees of freedom for U as calculated by the Welch-Satterthwaite formula ⁽¹⁾
 u_{cal} is the standard uncertainty of the calibration (1 sigma)
 u_{reg} is the standard error of the regression (1 sigma)

u_{reg} is given by the following well known equation for the uncertainty of a regression line ⁽²⁾.

$$u_{reg} = S_{reg} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2}} \quad (3)$$

where \bar{x} is the mean of the regression input time data
 n is the number of regression data points
 S_{reg} is the standard error for the regression (\sqrt{MSE})

An alternate form for equation 3 that is easier to compute is as follows:

$$u_{reg} = S_{reg} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{n S_x^2}} \quad (4)$$

where S_x is the standard deviation of the time data (x).

Substituting equation 4 into equation 2 gives the following expression for the total uncertainty of a characterized single cell DCVS as a function of time (X):

$$U \approx \pm t(1 - \alpha, v_{eff}) \sqrt{S_{reg}^2 \left(\frac{1}{n} + \frac{(x - \bar{x})^2}{n S_x^2} \right) + u_{cal}^2} \quad (5)$$

Equation 5 is used to estimate uncertainty at historical dates of interest, especially the current time. It also is used by some to predict the uncertainty of future performance.

Equation 5, of course, requires historical data on which to compute the various parameters. In its present form, it is difficult to use for planning purposes such as for estimating uncertainty performance assuming typical values for the parameters.

Equation 5 can be further simplified by considering the special case where historical data is collected in equally spaced time increments. In that case, S_x can be approximated by

$$S_x \approx \frac{a}{\sqrt{3}} \quad \text{where } 2a \text{ is the time period of the historical data}$$

Substituting this value for S_x into equation 5 gives:

$$U \approx \pm t(1 - \alpha, v_{eff}) \sqrt{S_{reg}^2 \left(\frac{1}{n} + \frac{3(\bar{x} - x)^2}{na^2} \right) + U_{cal}^2} \quad (6)$$

Equation 6 expresses the total uncertainty of a characterized single cell DCVS as a function of time (x) for the special case of n equally spaced calibration periods in a time period of 2a; that is the period $(\bar{x} - a)$ to $(\bar{x} + a)$. It is well suited for making uncertainty estimates based on typical DCVS performance rather than actual data.

For example, lets determine the time x for the next scheduled calibration and calculate the expected total uncertainty at the time just prior to recalibration. That value is the uncertainty maintained by the DCVS.

In the above model, the last calibration occurred at $X = \bar{x} + a$. The next calibration is scheduled for $x = \bar{x} + a + \frac{2a}{n}$ (since we have n calibrations in a 2a time period).

Substituting this last value of x into equation 6 gives the estimated uncertainty at which the standard is maintained.

$$U \approx \pm t(1 - \alpha, v_{eff}) \sqrt{\frac{S_{reg}^2}{n} \left(1 + 3\left(1 + \frac{2}{n}\right)^2 \right) + U_{cal}^2} \quad (7)$$

Equation 7 estimates the uncertainty at which a single cell DCVS is maintained as a function of the total number of calibration points n, the standard error of the corresponding regression (S_{reg}) and the standard uncertainty of the supporting calibrations (U_{cal}). It is useful for estimating the calibration interval required to maintain the desired uncertainty.

The above analysis can be expanded to allow a group of N DCVSs. For N standards equation 2 becomes

$$U = t(1 - \alpha, v_{eff}) \sqrt{\sum_1^N \frac{u_{reg_i}^2}{N^2} + u_{cal}^2} \quad (8)$$

where

$$u_{reg_i} = s_{reg_i} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{n s_{x_i}^2}} \quad (9)$$

Equation 8 is equivalent to equation 6 for multiple cell DCVSs. It is used to estimate the uncertainty of multi cell DCVSs at historical dates of interest, especially the current time. It also is used by some to predict the uncertainty of future performance.

For the special case of N independent DCVSs, each with n equally spaced calibrations in a time period of 2a, equation 8 simplifies to the following:

$$U \approx \pm K \sqrt{\frac{s_{pooled_reg}^2}{nN} \left(1 + 3\left(1 + \frac{2}{n}\right)^2\right) + u_{cal}^2} \quad (10)$$

where $s_{pooled_reg}^2$ is the pooled Standard Error for the N regressions.
 K is the coverage factor

In this case, $s_{pooled_reg}^2$ is the average Standard Error for the N regressions since all of the regressions have the same number of data points n. Coverage factor K replaced the t statistic in equation 10 since the degrees of freedom with multiple cells is relatively high. $K=2$ implies a 0.95 Confidence Factor.

Equation 10 is the multiple cell equivalent to equation 7. It is well suited for making uncertainty estimates on N independent DCVSs based on typical cell performance rather than actual data.

Now lets estimate the uncertainty of a single and multiple cell DCVS that have been characterized with a linear regression.

Assume a single cell DCVS will be calibrated at n regular intervals with an uncertainty of 0.1 ppm (using a Josephson Junction Array system) with 40 degrees of freedom. Further assume that it is characterized with a linear regression line having a regression standard error $s_{reg} = 0.14$ ppm (a typical value for data collected over two years or more).

Equation 7 or 10 can now be used to calculate the total uncertainty of a single cell or multiple cell DCVS as a function of the number of total calibrations n.

The resulting computations for the uncertainties U maintained by a single cell and a triple cell DCVS are given in Figure 2. For a single cell, this chart shows that 7 calibrations are required to maintain 0.3 ppm or better. This can be accomplished by calibrating every 6 months for at least 3 1/2 years or any other combination that totals seven calibrations. If the DCVS consists of a group of three cells, only 4 calibrations are

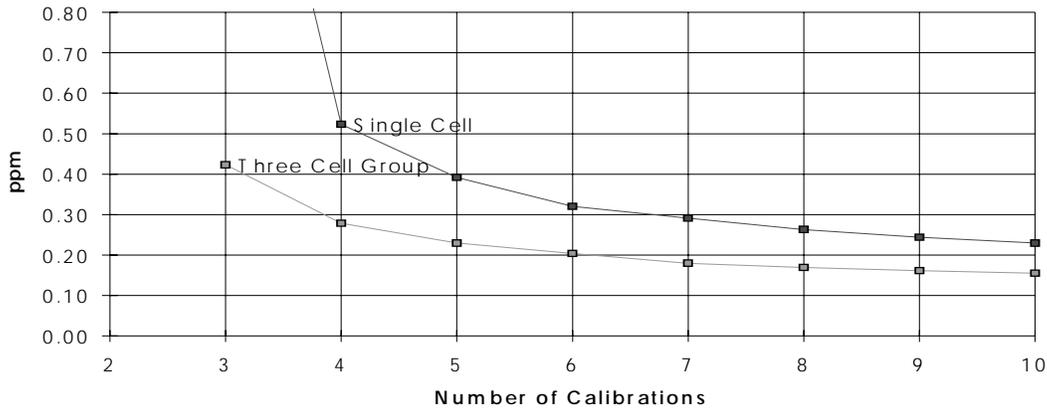


Figure 2. Required Number of Calibrations for a Given Uncertainty

required to maintain 0.3 ppm or better. This can be done by calibrating the group twice a year for two years.

After the desired uncertainty has been achieved, the next calibration must be done at its normal calibration interval ($2a/n$). After that, however, the calibration intervals can be gradually increased since the total number n is increasing. Equation 5 (for a single cell) or 8 (for a multiple cell) can be used to estimate when that calibration is required.

One might observe that 0.3 ppm can be obtained by calibrating a group of three cells for 4 periods, or a single cell for 7 periods. The former requires 12 calibrations and the later only 7. This suggests that it is cheaper to use only a single cell standard. Actually, a single cell reference may not be sufficient for several reasons.

The analysis presented so far in this paper is developed around a simplified model. The output voltage of the dc standard is assumed to be a function only of time provided it is operated within a specified temperature window. This simplified model may be sufficient when performance of a few ppm is involved, but it loses its appropriateness when dealing with fractional ppm. The following section will expand the scope of the model for the DCVS and discuss each component.

Expanded Model

In order to be suitable for fractional ppm performance, the model for a dc reference standard should include the effects of nonlinear drift, temperature, pressure and seasons. Whereas each of these components typically contributes little with regard to the

manufacture's specification, they can have a significant effect on performance of 1 ppm or better. We will examine each of these effects in the following paragraphs.

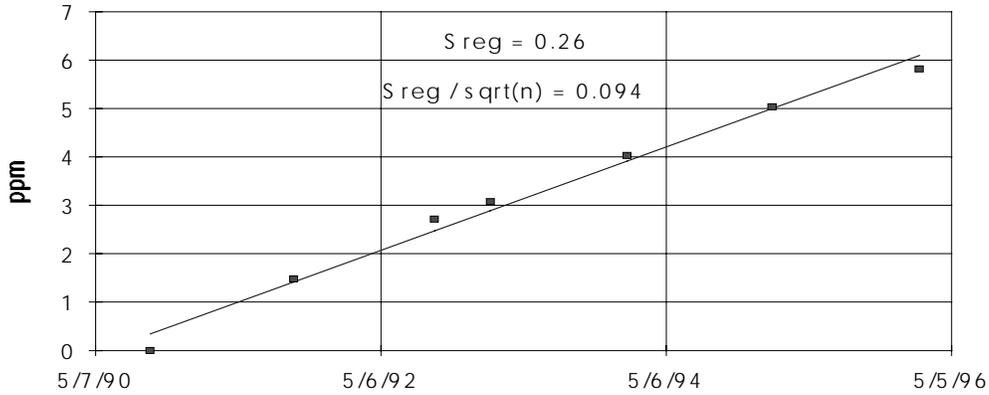


Figure 3. Linear Regression of Non Linear Data

Non linearity with time: Many solid state dc reference standards drift nearly linearly over long time periods. The analysis presented earlier in this paper relates to linear drift. The drift rate on some, however, are nonlinear, especially if the observed periods are longer than 2 or 3 years. An example of this is given in Figure 3.

The relatively poor fit of the linear regression is indicated by a relatively high value for the regression standard error S_{reg} (0.26 ppm in this case). Earlier analysis in this paper assumed a S_{reg} of 0.14 ppm.

Sometimes an improvement can be made in the fit and uncertainty performance simply by omitting one or more of the earliest data points. This reduces S_{reg} which reduces uncertainty; however, reducing n tends to increase uncertainty. Equations 5 through 10

show that uncertainty is proportional to the factor $\frac{S_{reg}}{\sqrt{n}}$. Therefore, when early data points are removed, this term should be tracked and minimized.

For example, let's attempt to improve the fit and reduce the uncertainty of the regression given in Figure 3 by removing one or more of its early data points. Figure 4 shows how

S_{reg} and $\frac{S_{reg}}{\sqrt{n}}$ vary with the number of remaining data points. Both parameters are

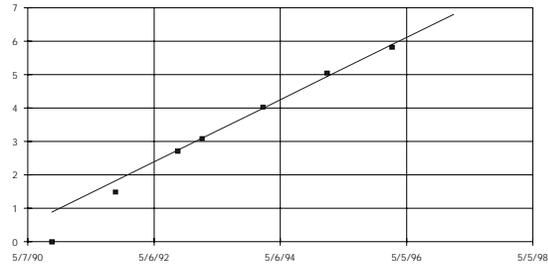
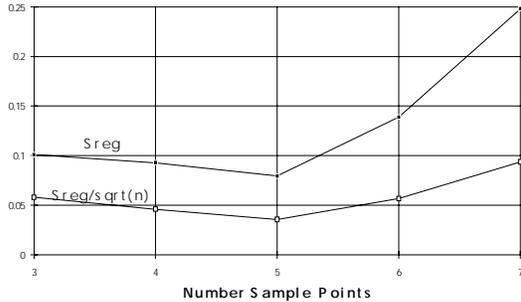


Figure 4. Uncertainty Parameter Variation Figure 5. Effect of Truncated Data

minimized when the first two data points are removed. Figure 5 shows the resulting representation of the data. The net effect is that S_{reg} is reduced from 0.24 ppm to 0.08 ppm and the noise parameter is reduced from 0.094 ppm to 0.036 ppm.

Another method for coping with nonlinearity is to linearize the relationship between the data and time by transforming the time variable x . Generally, it is possible to linearize by transforming one or both of the variables, however, if the expressions for uncertainty are to remain valid, only the time variable can be transformed ⁽³⁾.

The type of transformation depends on the shape of the data. For a nonlinearity that tends to decrease the magnitude of the data with time, as in Figure 3, transforms including $x' = \sqrt{x}$, $x' = \log(x)$, or $x' = \frac{1}{n}$ are recommended in the literature ⁽⁴⁾. However, we have found that the transformations $x' = \log(x - d)$, and $x' = \sqrt{x - d}$ are the most useful for nonlinearities commonly encountered in dc voltage standards. The constant d is usually a number corresponding to a date less than the earliest data by several months.

Each of the above transformations was applied to the non linear data of Figure 3. The following tabulates the resulting values for the important uncertainty parameters S_{reg} and

$$\frac{S_{reg}}{\sqrt{n}}$$

Transformation	S_{reg}	$\frac{S_{reg}}{\sqrt{n}}$
none (linear with 7 data pts)	0.25 ppm	0.094 ppm
truncated linear 5 data pts	0.08	0.036
$x'=\log(x)$	0.23	0.087
$x'=\sqrt{x}$	0.24	0.09
$x'=\log(x-30500)$	0.037	0.014
$x'=\sqrt{x-32600}$	0.036	0.0137

Table 1. Effect of Variable Transformation on Uncertainty Parameters

Table 1 shows that for the non linear data given in Figure 3, the transformations

$x'=\log(x-d)$ and $x'=\sqrt{x-d}$ both reduce S_{reg} from 0.25 to 0.037 and $\frac{S_{reg}}{\sqrt{n}}$ from 0.094 ppm to 0.014 ppm. The uncertainty of the transformed regression is improved over that of the normal linear regression by a factor of nearly 7. That is well worth the added complexity of the uncertainty calculations.

Temperature Effects: Modern DCVSs are designed to operate across a wide temperature range without degrading their performance. Typically, these units operate at $23\pm 5^\circ C$ within their specified performances. In this case, however, we are interested in changes significantly smaller than the usual stability specifications. Therefore, it is necessary to consider the effects of temperature variations around the calibration temperature.

If the temperature coefficient is specified as $TC \text{ ppm}/^\circ C$, and the observed temperature differs from the temperature at which the DCVS was calibrated at by $\delta^\circ C$, then the change in the DCVS output voltage is bounded by the product $TC * \delta$. Actually, TC is a random variable that can reasonably be modeled as having a normal distribution with a standard deviation $S_{tc} = \frac{TC}{\sigma}$ where $\sigma=2$ if the DCVS is specified at a 95% Confidence Factor (CF) or $\sigma=2.6$ if specified at a 99% CF. This gives the standard uncertainty for temperature effect contributions as

$$u_{tc} = \frac{TC}{\sigma} \delta \quad (11)$$

The uncertainty term given by equation 11 can be minimized simply by minimizing δ ; that is, keep the temperature of the DCVS near the temperature at which it is calibrated. If the temperature difference is significant, then the uncertainty term of equation 11 should be applied.

Pressure/Altitude Effects: Sensitivity to atmospheric pressure is not a new phenomena for reference standards. This effect is well known for Thomas type 1 ohm resistors, for

example. It may come as a surprise, however, that some zener type DCVSs also exhibit sensitivity to pressure.

Measurements made on a limited sample of DCVSs show pressure sensitivities ranging from near zero to approximately -0.06 ppm/1000 foot altitude. The data collected is too limited to quantify the distribution of units within these bounds. Units seem to be grouped at -0.06, -0.03 and near 0 ppm/1000 foot altitude.

The standard uncertainty for pressure sensitivity can be modeled as a pressure sensitivity times the altitude.

$$u_p = ph \tag{12}$$

where p is the pressure sensitivity in ppm/1000 ft
and h is the height (altitude) in 1000s of feet

The effects of altitude sensitivity can be minimized or eliminated by at least two methods. The classical method is to characterize the standard and apply a correction term. This reduces the uncertainty of the pressure effect to the uncertainty of the correction term. The uncertainty term given in equation 12 then becomes the standard uncertainty of the correction term rather than that for the larger pressure sensitivity.

Another approach is to utilize an on site calibration service (MAP). In this case, the calibration supplier is responsible for compensating for the pressure sensitivity of his standard. This method essentially eliminates pressure uncertainty.

Seasonality: Seasonality refers to a cyclic variation in the output of the DCVS that repeats annually. Figure 8 is a representation of such a unit.

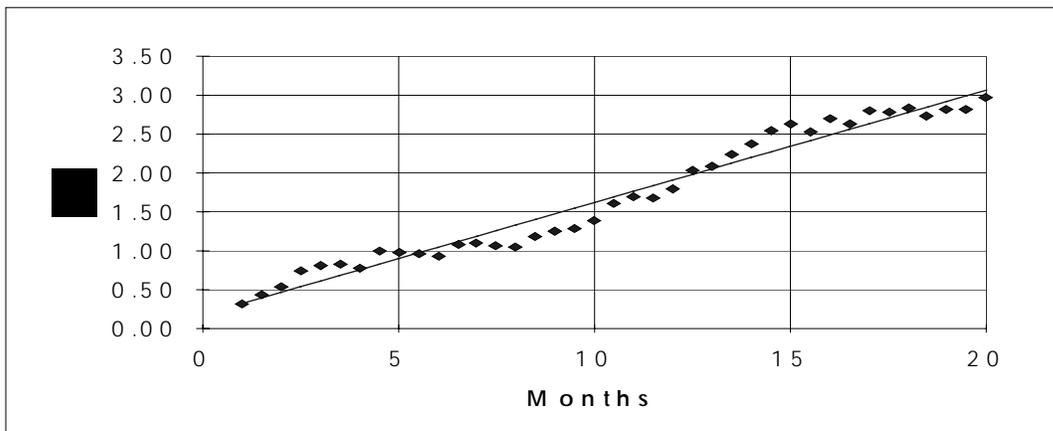


Figure 8. Example of a DCVS with Seasonal Variations

The above example shows a seasonal effect of approximately 0.2 ppm peak. Many units show little or no effect. Supporting data is very limited since the DCVS must be monitored continuously with a high precision measurement system; therefore, it is difficult to estimate the distribution of this effect. Based on the limited data we have collected, I believe that it can be conservatively estimated by a standard uncertainty of 0.12 ppm peak. That is

$$u_s \approx 0.12 \text{ ppm} \quad (13)$$

where u_s is the estimated standard uncertainty (1 sigma) for the seasonal effect

The cause for this effect has not been established; however, the best hypotheses is that water vapor in the environment causes leakage across critical internal elements. Most units exhibiting this phenomena reside in laboratories with poor or no humidity control.

Seasonal effects can cause an understatement of your DCVS uncertainty if it is unaccounted for. For example, many labs calibrate their standards at about the same time every year. Since the seasonal effect is synchronous with the calendar, the calibration will occur at the same phase of this effect every year. Its contribution to the performance of the DCVS may never be observed or accounted for. If the calibration is done asynchronously, then over a period of time all phases of this effect will be observed. This will automatically be reflected as an increased value in the standard error of the regression line S_{reg} .

Multiple standards can be helpful for detecting and quantifying seasonal effects.

Averaging n standards will reduce the seasonal effect by approximately $\frac{1}{n}$ since their magnitude and polarity are not highly correlated. Therefore, if each unit is compared to the group average, its seasonality can be observed and its magnitude estimated. This will allow the user to eliminate poor performers or to improve the estimation of the standard error for u_s given by equation 13.

Expanded Model: Earlier in this paper, equations 5-10 were developed for total DCVS uncertainty using a simple model reflecting only the effects of time. All other effects were assumed to be negligibly small. These equations now will be expanded to include the uncertainty terms developed in this section for temperature, pressure, and seasonal effects.

When the uncertainty terms for the expanded effects are combined with equation 5, it becomes the following:

$$U \approx \pm K \sqrt{S_{reg}^2 \left(\frac{1}{n} + \frac{(\bar{x} - \bar{x})^2}{n S_x} \right) + u_{cal}^2 + u_{TC}^2 + u_p^2 + u_s^2} \quad (14)$$

Equation 14 is an improved model for the total uncertainty of a single DCVS. It can be used when historical data is available for the unit. The factor $t(1-\alpha, v_{eff})$ in equation 5 was changed to coverage factor K since the effective degrees of freedom in the improved equation will usually result in K being near 2 for a 0.95 confidence level or near 2.6 for a 0.99 confidence level.

Similarly, the approximation for a single DCVS given by equation 7 becomes:

$$U \approx \pm K \sqrt{\frac{S_{reg}^2}{n} \left(1 + 3 \left(1 + \frac{2}{n} \right)^2 \right) + u_{cal}^2 + u_{TC}^2 + u_p^2 + u_s^2} \quad (15)$$

Equation 15 can be used to estimate total uncertainty when actual data has not been collected. It assumes n equally spaced calibrations.

The uncertainty of a group of N single cell DCVSs as given by equation 8, changes to the following when the expanded uncertainty contributors are included:

$$U = K \sqrt{\frac{u_{pooled_reg}^2}{N} + u_{cal}^2 + \frac{u_{pooled_TC}^2}{N} + u_{pooled_p}^2 + \frac{u_{pooled_s}^2}{N}} \quad (16)$$

where $u_{pooled_y}^2$ refers to the pooled value of u_y^2

Each standard uncertainty squared term was divided by N since the terms are assumed to be independent. This is not the case for the pressure term, however, since the pressure sensitivity for many units will be similar. Therefore, the pressure sensitivity will not be reduced with multiple units.

Equation 16 can be used to calculate the total uncertainty for a N-cell DCVS when historical data is available.

Similarly, equation 10 expands to the following:

$$U \approx \pm K \sqrt{\frac{S_{pooled_reg}^2}{nN} \left(1 + 3 \left(1 + \frac{2}{n} \right)^2 \right) + u_{cal}^2 + \frac{u_{pooled_TC}^2}{N} + u_p^2 + \frac{u_{pooled_s}^2}{N}} \quad (17)$$

This equation is an approximation for multiple (N) units for which n equally spaced historical data are assumed.

Examples

We will illustrate the applications of the above uncertainty equations with two scenarios as follows:

Scenario 1: A laboratory has a need for maintaining 10 VDC at an uncertainty of ± 0.3 ppm with a 95% confidence level. Its metrologist is planning on utilizing 3 zener type standards with a TC of ≤ 0.05 ppm/ °C and pressure sensitivity of ≤ 0.03 ppm/1000 ft. He understands these units typically have a regression standard error $s_{reg} \leq 0.14$ ppm. The laboratory is located at an elevation of 5000 feet. Its environment is maintained at 23 ± 3 °C. He plans to send his units out for calibration at an uncertainty of ± 0.1 ppm in a lab maintained at 23 ± 0.6 °C. Can 0.3 ppm be maintained? How many calibrations are required?

Equation 17 is relevant for this scenario. $N=3$, $s_{reg} \leq 0.14$ ppm, and $u_{cal} = 0.1$. u_{TC} can be calculated from equation 8 as follows:

$$u_{TC} = \frac{TC}{\sigma} \delta = \frac{0.05}{2}(3) = 0.075 \text{ ppm}$$

where δ is set at 3 since the laboratory will function across the full temperature range 23 ± 3 °

Pressure standard uncertainty at 5000 feet is calculated from equation 12

$$u_p = 0.03 * 5 \text{ ppm} = 0.15 \text{ ppm}$$

Seasonal effects are unknown, therefore, we will use the estimation of equation 10.

$$u_s \approx 0.12 \text{ ppm}$$

At the 95% confidence level, the coverage factor $K=2$.

Now inserting all the above values into equation 17 gives the following estimation:

$$U \approx 2 \sqrt{\frac{0.0065}{n} \left(1 + 3 \left(1 + \frac{2}{n} \right)^2 \right)} + 0.0392 \text{ ppm} \quad (18)$$

Equation 18 is plotted in Figure 9 for several values of n. In this case, the total uncertainty, although low, does not approach the desired 0.3 ppm level due to the relatively large contributions of temperature, pressure and seasonal effects.

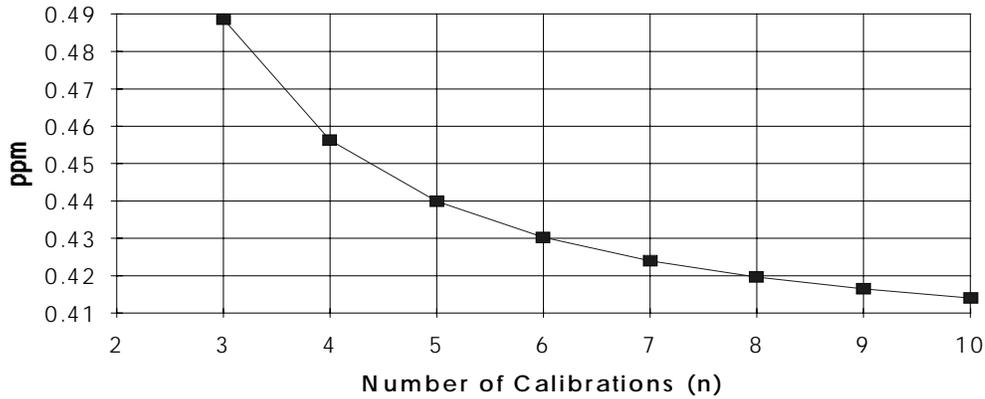


Figure 9. Uncertainty for Scenario 1

Scenario 2: This scenario is the same as Scenario 1 except let the calibration be an on-site 0.1 ppm MAP. Also assume that intercomparison data is available on the three cells which show that one cell has a standard seasonal uncertainty $U_s \approx 0.12 \text{ ppm}$.

In this case there is no pressure effect since MAP's reference cells have negligible pressure sensitivity. Also, since only one of the three cells show a seasonal effect of 0.12 ppm, $U_{pooled_p}^2 = 0.0048$. Inserting these values into equation 17 gives:

$$U \approx 2 \sqrt{\frac{0.0065}{n} \left(1 + 3 \left(1 + \frac{2}{n} \right)^2 \right)} + 0.014 \text{ ppm} \quad (19)$$

A plot of U versus the number of equally spaced calibrations (n) is given in Figure 10. It shows that the 0.3 ppm uncertainty goal can be reached after 5 or more equally spaced calibrations. If data is collected over a two year period, a calibration is required every 5 months. A calibration every 7 months is required if collected over a three year period.

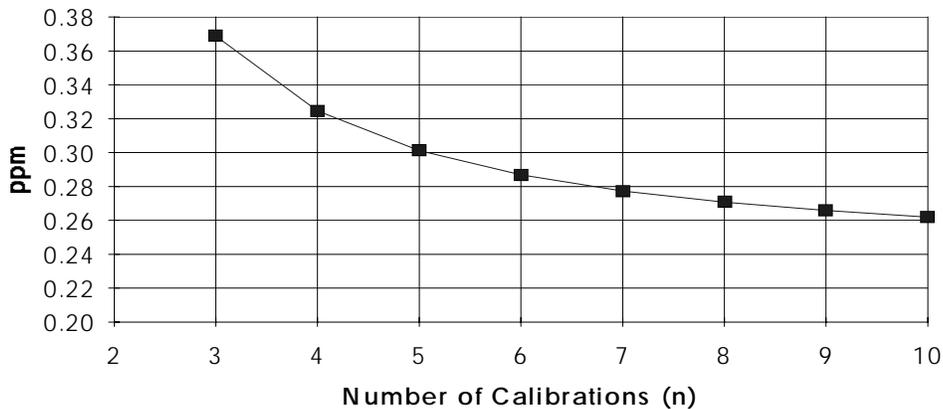


Figure 10. Uncertainty for Scenario 2

Conclusions

This paper describes how to establish and maintain dc reference standards in your laboratory at an uncertainty of 0.3 ppm or better. It requires observing and minimizing the effects of terms often disregarded in uncertainty budgets. These include nonlinear, seasonal, and pressure/altitude effects. The use of multiple standards is suggested to reduce random errors, temperature sensitivity, and to detect and bound any seasonal effect.

Calibration of the standards can best be done utilizing a commercial MAP-type on-site calibration service. Calibration uncertainties of 0.1 ppm is the norm for services using Josephson Junction Array Systems. On-site calibration also eliminates system down time, shipping effects, and pressure sensitivity error since the laboratory's dc reference standards never leave their environment.

Examples discussed in this paper show 5 or more equally spaced calibrations are required on three or more DCVSs to achieve and maintain 0.3 ppm uncertainty. If this data is collected over a three year period, calibrations are required every 7 months. After 6 calibrations have been completed, it may be possible to increase the calibration period and still maintain the uncertainty goal. Equations 14 or 16 can be used to calculate the next required date of calibration.

This analysis is based on practical values and reasonable assumptions. We believe that most laboratories can realize 10 VDC at ± 0.3 ppm uncertainty or better without the complexity and cost of operating and maintaining a J J Array.

References

¹ Taylor, Barry N. and Kuyatt, Chris E., "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results", NIST Technical Note 1297, p9

² Neter, John, Wasserman, William, Kutner, Michael H., "Applied Linear Statistical Models", Third Edition, 1990, IRWIN, pp71-77

³ Ibid., p. 142

⁴ Ibid, p. 143